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STUDY OF BATCH DRYING

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Abstract—This paper proposes two mathematical models giving possibility to study the mechanism of batch drying. The first part deals with the influence of the changing properties of the drying medium and the second considers the influence of the gradients in the cross section. The proposed models are suitable for study of drying of finite plate under changing and non-linearized sorption properties. The numerical solutions are illustrated by examples calculated by computer.

NOMENCLATURE

a, temperature conductivity [m^2/h];
c, specific heat [kcal/kg °C];
c, concentration [kg/m^3];
e, measure [m];
f, factor defined by equation (7) [dimensionless];
h, measure [m];
i, enthalpy [kcal/kg];
k, over-all heat-transfer coefficient [$kcal/m^2$ h °C];
P, partial pressure [Hg mm];
q, heat flux [$kcal/m^2$ h];
r, latent heat [kcal/kg];
t, temperature [°C];
x, moisture content of drying medium [kg moisture/kg dry air];
y, co-ordinate [m];
z, co-ordinate [m];
D, diffusion coefficient [m^2/h];
F, surface [m^2];
G, mass of moist body [kg];
N, rate of drying [kg/m^2 h];
X, moisture content of the moist body [kg/moisture/kg dry material];
α, heat-transfer coefficient [$kcal/m^2$ h °C];
ε, limit of the error [dimensionless];
λ, thermal conductivity [kcal/m h °C];
ρ, density [kg/m^3];
σ, coefficient of the evaporation [kg/m h];
τ, time [h];
φ, sign of function defined by equation (6);
γ, sign of function defined by equation (7);
ψ, ratio of the recirculation [dimensionless];
Δ, measure of the step [m or h];
ν, $= \dfrac{t}{t_0}$, dimensionless temperature;
ξ, $= \dfrac{c}{c_0}$, dimensionless concentration;
Bi, Biot number;
Fo, Fourier number;
Le, Lewis number;
Lu, Luikov number.

Subscripts

e, equilibrium value;
$fé$, reference to the surface;
G, gas;
kd, conductive;
kr, critical;
kv, convective;
L, reference to the drying medium;
m, mass;
na, moist material;
nL, wet air;
0, reference to the zero temperature [°C];
PWG, vapor under constant pressure;
q, heat;
szt, volume of dry material;
tel, saturation;
W, wet.

INTRODUCTION

THE BATCH drying is a chemical operation frequently used in the chemical industry especially for materials which need a long drying period, e.g. brick, wood etc. At the batch drying the moisture content profiles and the distribution of the temperature in the moist body generally depend on the time, because the temperature and the moisture content of the medium as well as the rate of the mass and heat transfer are changing.

The present paper gives methods to find temperature profiles and the moisture content field, so two models are proposed for batch drying.

These models have different neglect namely (i) model one neglects the influence of the concentration and the thermal gradient being in the moist body in the cross-section; and (ii) model two neglects the changes of the properties of the surrounding medium.

The results obtained are demonstrated by numerical examples using computer.

MODEL ONE

In a previous paper [1] we neglected the changes of the properties—the temperature and moisture content—of the surrounding medium. Sometimes this is possible, e.g. using a respectable mass of the drying medium, but sometimes is not.

In the first part of this paper we develop a mathematical model considering these changes. In the numerical example we shall see that remarkable moisture and temperature gradients arising from these changes will be carried out in the moist body in the direction of the flow of drying medium. These gradients are able to damage the structure of the body by the increased stress.

Analysis

Considering an elementary section of the drying equipment (Fig. 1) the heat and mass balances can be written.

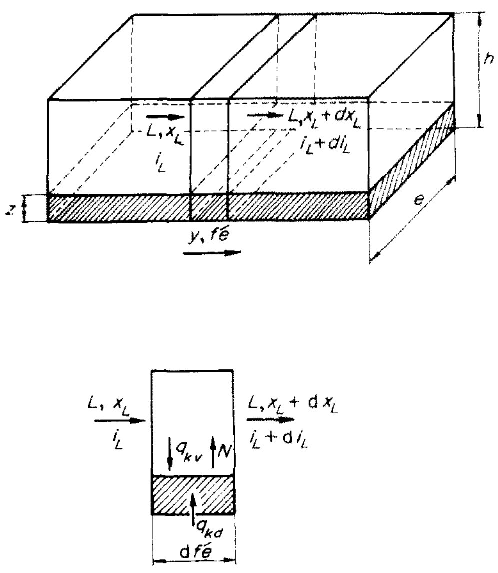

FIG. 1. Sketch of the basic model.

Mass balance for the moisture in the body:

$$\mathrm{d}G_{sz}X = N\mathrm{d}f_{\acute{e}}\,\mathrm{d}\tau + \mathrm{d}G_{sz}\left(X + \frac{\partial X}{\partial \tau}\mathrm{d}\tau\right),$$

where

$$\mathrm{d}G_{sz} = z\,\mathrm{d}f_{\acute{e}}\rho_{szt} = \mathrm{d}V\rho_{szt},$$

therefore

$$-\frac{\partial X}{\partial \tau} = \frac{1}{z\rho_{szt}}N. \tag{1}$$

Mass balance for the moisture in the surrounding medium

$$Lx_L\,\mathrm{d}\tau + eh\,\mathrm{d}y\rho_G x_L + N\,\mathrm{d}f_{\acute{e}}\,\mathrm{d}\tau = L\left(x_L + \frac{\partial x_L}{\partial f_{\acute{e}}}\mathrm{d}f_{\acute{e}}\right)\mathrm{d}\tau + eh\,\mathrm{d}y\rho_G\left(x_L + \frac{\partial x_L}{\partial \tau}\mathrm{d}\tau\right).$$

Where the groups on the left-hand of the equation mean the mass of the moisture carried into the section by the drying medium in the elementary time; the mass of the moisture being in the section at the beginning of the elementary term, and the mass of the moisture evaporated into the section during the elementary term. Forthcoming we shall use this order.

Because

$$\mathrm{d}f_{\acute{e}} = e\,\mathrm{d}y,$$

therefore

$$L\frac{\partial x_L}{\partial f_{\acute{e}}} + h\rho_G\frac{\partial x_L}{\partial \tau} = N. \tag{2}$$

The heat balance for the surrounding medium:

$$Li_L\,\mathrm{d}\tau + eh\,\mathrm{d}y\rho_G i_L + Ni_F\,\mathrm{d}f_{\acute{e}}\,\mathrm{d}\tau = L\left(i_L + \frac{\partial i_L}{\partial f_{\acute{e}}}\mathrm{d}f_{\acute{e}}\right)\mathrm{d}\tau, + eh\,\mathrm{d}y\rho_G\left(i_L + \frac{\partial i_L}{\partial \tau}\mathrm{d}\tau\right) + \alpha_{kv}(t_L - t_F)\,\mathrm{d}f_{\acute{e}}\,\mathrm{d}\tau.$$

On the right-hand the third group is the heat transported towards the body. As well known:

$$i_L = (c_{PL} + x_L c_{PWG})t_L + r_0 x_L,$$

so

$$\mathrm{d}i_L = c_{nL}\,\mathrm{d}t_L + c_{PWG}t_L + r_0)\mathrm{d}x_L,$$

where

$$c_{nL} = c_{PL} + x_L c_{PWG}$$

the heat capacity of the moist air. Considering the enthalpy of the drying medium on the evaporating surface:

$$i_F = r_0 + c_{PWG}t_F$$

and using (2) after reduction we obtain:

$$-Lc_{nL}\frac{\partial t_L}{\partial f_{\acute{e}}} = \alpha_{kv}(t_L - t_F) + Nc_{PWG}(t_L - t_F) + h\rho_G c_{nL}\frac{\partial t_L}{\partial \tau}. \tag{3}$$

The heat balance for the moist body:

$$dG_{sz}i_{na}+(q_{kd}+q_s)\,df_é\,d\tau = dG_{sz}\left(i_{na}+\frac{\partial i_{na}}{\partial\tau}d\tau\right)+N\,df_é i_F\,d\tau.$$

We suppose a linear relation between the heat capacity of the moist body and the moist content:

$$i_{na}=(c_{sz}+Xc_W)t_{na},$$

and

$$di_{na}=c_{na}\,dt_{na}+c_W t_{na}\,dX,$$

where

$$c_{na}=c_{sz}+Xc_W.$$

Further it should be assumed that the temperature and the moisture content are constant in direction z:

$$t_{na}=t_F.$$

Using these equations and the heat balance for the boundary layer [1] can be written:

$$z\rho_{szt}c_{na}\frac{\partial t_F}{\partial\tau}=\alpha_{kv}(t_L-t_F)+k_{kd}(t_{kd}-t_F) -N(r_0+c_{PWG}t_L-c_W t_F). \quad (4)$$

In order to complete this system (1)–(4) we assume the following relations:

$$N=\sigma(x_F-x_L)f, \quad (5)$$

where

$$x_F=\varphi(t_F), \quad (6)$$

and

$$f=\gamma(X). \quad (7)$$

It is advisable to determine the later relation by measurement, although there is possibility to use some approaching ideas, too. Finally we do repeat that we have supposed there was not thermal either concentration gradient in the moist body rectangular to the flow of the drying medium.

The initial and boundary conditions

In order to get the solution we have to know the following conditions:

$$X(f_é,0)=X_0,$$
$$x_L(0,\tau)=x_{Lb},$$
$$x_L(f_é,0)=x_{Lb},$$
$$t_L(0,\tau)=t_{Lb},$$
$$t_L(f_é,0)=t_{Fo},$$
$$t_F(f_é,0)=t_{Fo}.$$

Numerical solution

We can solve this system in the following way:

$$\frac{\partial x_L}{\partial f_é}(f_é,\tau)=\frac{x_L(f_é+\Delta_{fé},\tau)-x_L(f_é,\tau)}{\Delta_{fé}}, \quad (8)$$

so considering (2) we can obtain:

$$x_L(f_é+\Delta_{fé},\tau)=x_L(f_é,\tau)+\Delta_{fé}\left[N(f_é,\tau)-h\rho_G\frac{\partial x_L}{\partial\tau}(f_é,\tau)\right] \quad (9)$$

where

$$\frac{\partial x_L}{\partial\tau}(f_é,\tau)=\frac{x_L(f_é,\tau+\Delta_\tau)-x_L(f_é,\tau)}{\Delta_\tau} \quad (10)$$

$$N(f_é,\tau)=\sigma[x_F(f_é,\tau)-x_L(f_é,\tau)]f(f_é,\tau) \quad (11)$$

$$x_F(f_é,\tau)=\varphi[t_F(f_é,\tau)]. \quad (12)$$

The other equations can be transformed into difference equations in the same way.

While solving numerically, sufficiently increment size was used to achieve a desired accuracy.

As an illustration a numerical example is carried out with the following dates:

$X_0 = 0{\cdot}25$ kg moisture/kg dry material
$x_{Lb} = 0{\cdot}04$ kg moisture/kg dry air
$t_{Lb} = 120\,°C$
$t_{Fo} = 20\,°C$
$\alpha_{kv} = 20$ kcal/m^2 h °C
$\sigma = 80$ kg/m^2 h
$y_{max} = 2{\cdot}5$ m
$z = 0{\cdot}06$ m
$e = 0{\cdot}12$ m
$h = 0{\cdot}1$ m
$L = 144$ kg/h
$c_{nL} = 0{\cdot}25$ kcal/kg °C
$c_{sz} = 0{\cdot}21$ kcal/kg °C
$c_W = 1$ kcal/kg °C
$\rho_{szt} = 2000$ kg/m^3
$\rho_G = 0{\cdot}9$ kg/m^3
$X_{kr} = 0{\cdot}11$ kg moisture/kg dry material
$X_e = 0{\cdot}01$ kg moisture/kg dry material.

The drying medium is air and the moist body is the row of crude bricks of which properties are obtained from the literature [2]. The results of the numerical example are shown on Figs. 2–11. On Figs. 2–6 the independent variable is the place co-ordinate and the parameter is the time. On Figs. 7–11 the situation is just opposite.

Considering Figs. 2 and 7 showing the change of the moisture content in the body we can find a respectable moisture gradient along its length, which may be increased by the intensification of drying process causing additional stresses.

It is seen the distribution of the temperature in the drying medium (Figs. 3 and 8) and temperature field along the length of the row of bricks (Figs. 4 and 9).

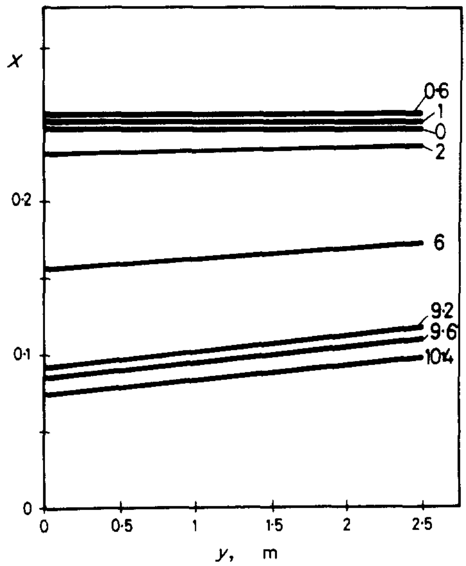

FIG. 2. The distribution of the moisture content of the moist body vs. length in varying time.

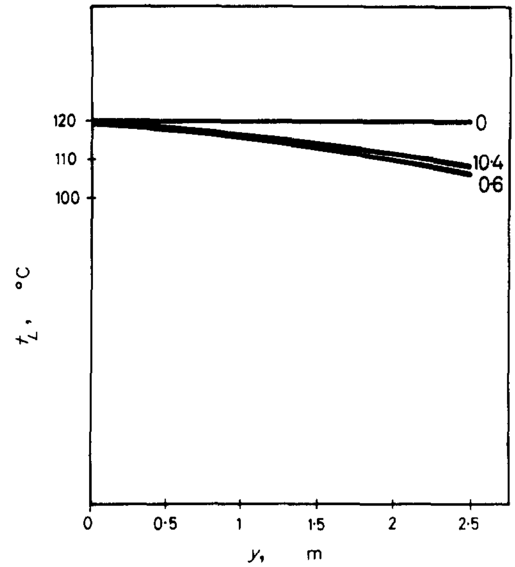

FIG. 3. The distribution of the temperature of the drying medium vs. length of the moist body in varying time.

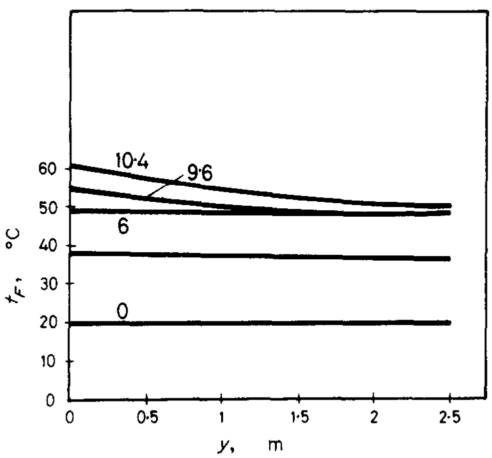

FIG. 4. The distribution of the temperature of the moist body vs. length in varying time.

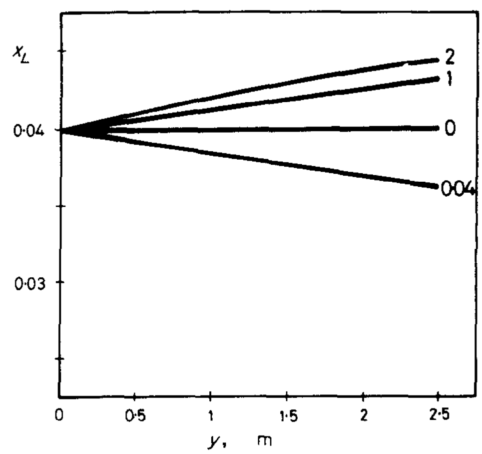

FIG. 5. The distribution of the humidity of the drying medium vs. length of the moist body in varying time.

The distribution of the moisture contents in the drying medium is illustrated on Figs. 5 and 10 showing the condensation of the moisture from the air at the beginning of the drying period.

These results are in comparison with the results of a simpler, lump model mentioned earlier [1]. On Fig. 7 we can see the deviation in the drying time required to achieve a stipulated moisture content is about 20 per cent, e.g. at $X = 0{\cdot}175$, $\tau_{y=0} = 5$ h and $\tau_{y=2.5} = 6$ h.

The total or partial recirculation of the drying medium is often used at the batch drying. The model is suitable to describe this process, too, being changed only the boundary conditions. The necessary changes are the following:

$$x_L(0,\tau) = x_L(0,0) \quad \text{if} \quad \tau < \tau_b,$$

$$x_L(0,\tau) = \psi x_L(F_e, \tau-\tau_b) + (1-\psi)x_L(0,0) \quad \text{if} \quad \tau \geqslant \tau_b,$$

and

$$t_L(0,\tau) = t_L(0,0) \quad \text{if} \quad \tau < \tau_b,$$

$$t_L(0,\tau) = \psi t_L(F_e, \tau-\tau_b) + (1-\psi)t_L(0,0) \quad \text{if} \quad \tau \geqslant \tau_b$$

where the $\psi 0 \leqslant \psi \leqslant 1$ is ratio of the recirculation. $\psi = 1$ means total recirculation (τ_b is the time of the recirculation).

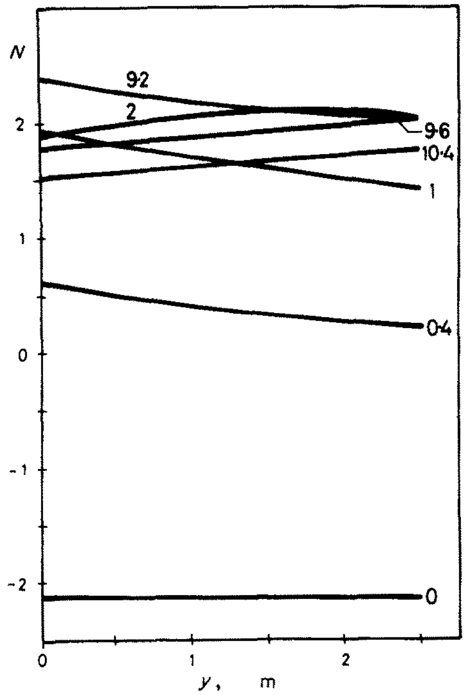

FIG. 6. The distribution of the drying rate vs. length of the moist body in varying time.

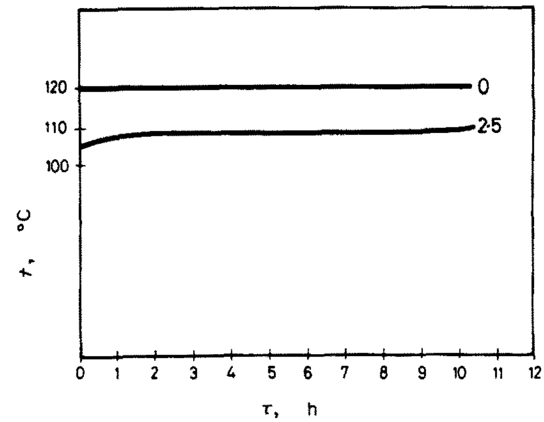

FIG. 8. Change in temperature of the drying medium, parameter y.

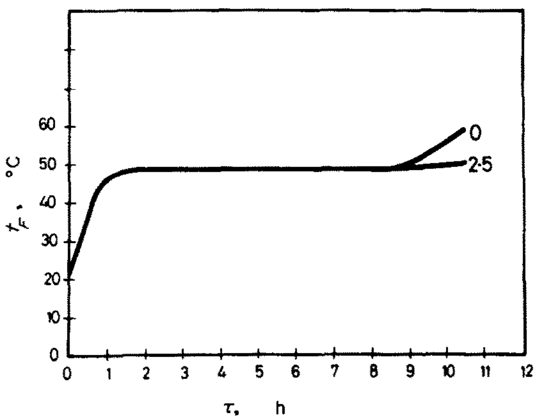

FIG. 9. Change in temperature of the moist body, using a constant parameter y.

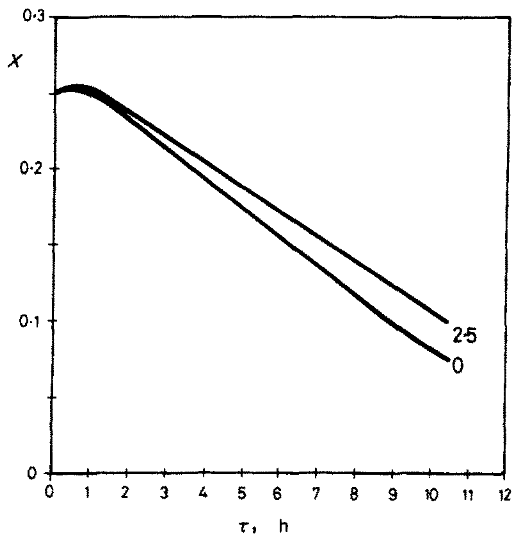

FIG. 7. Change in moisture content of the moist body, using a constant parameter y.

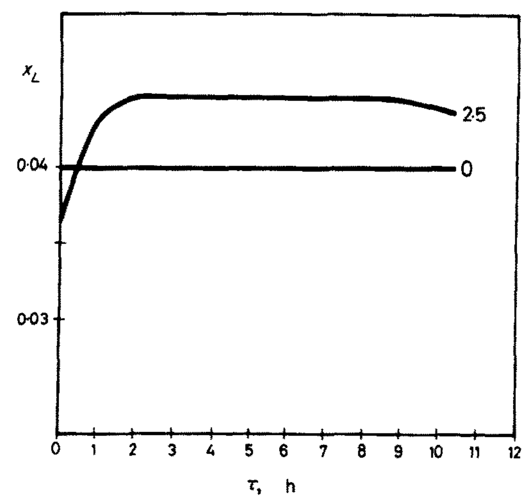

FIG. 10. Changes in the humidity of the drying medium, using a constant parameter y.

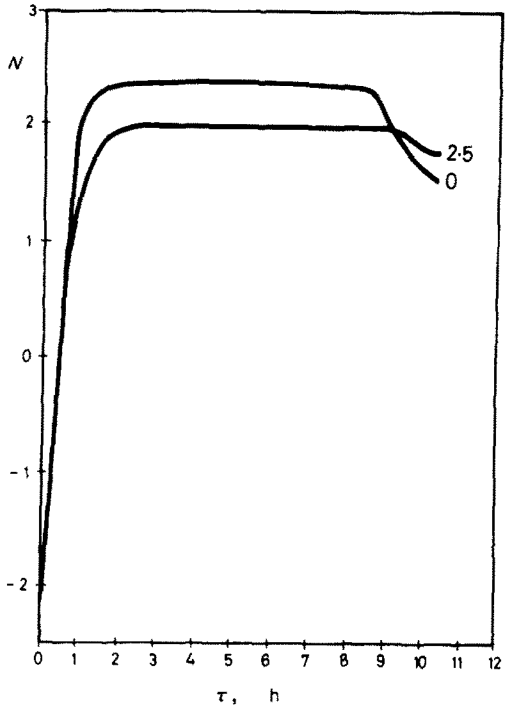

FIG. 11. Changes in the drying rate, using a constant parameter y.

MODEL TWO

At the earlier model the coefficient of the heat conductivity and the coefficient of the mass conductivity were supposed infinite in the cross direction and we have studied only the change of the properties of the drying air and their influence to the drying process.

Now, we take in account the opposite case. We neglect the changes in the drying medium properties and consider the distributions of the temperature and the moisture content in the cross-section of the moist body. We assume that the temperature and the moisture content are constant in the bulk of the surrounding medium.

We consider the heat transport caused by the moisture diffusion, too, which is a cross effect from the view of the heat transport by conduction. We assumed the heat capacity, the density and the coefficient of the heat and mass conductivity of the moist body are constant. In this second model we used the dimensionless form of the equations which would have been artificial in the first one.

In connection with the numerical solution we give an iterative method which is usable for similar complicated boundary conditions.

Analysis

As the first step the heat and mass balance for the concentration and thermal boundary layer can be written assuming the ratio of their thicks are nearly equals. In case of air–water system it is generally satisfied.

Considering Fig. 12 the mass balance for the moisture being in the drying medium may be written:

$$N = \sigma(x_F - x_L). \tag{13}$$

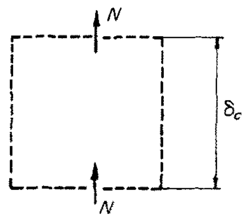

FIG. 12. Sketch for the material balance of moisture at the diffusion boundary layer of the drying medium.

The heat balance can be expressed according to Fig. 13:

$$Ni_F + q_{kv} = q_s + Ni_L$$

namely

$$q_s = (\alpha_{kv} - c_{PWG}N)(t_L - t_F). \tag{14}$$

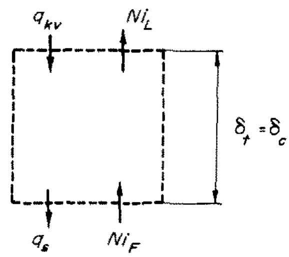

FIG. 13. Sketch for the heat balance at the thermal boundary layer of the drying medium.

Then let us write the heat and mass balance for the moist body assuming that c, λ, ρ and D are constant and the contraction of the moist body is negligible.

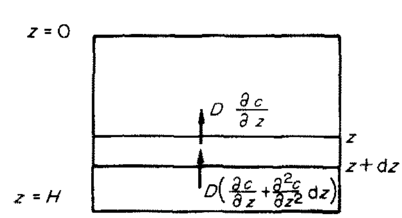

FIG. 14. Sketch for the material balance of the moisture in the moist body.

Considering Fig. 14 we can write after reducing:

$$\frac{\partial c}{\partial \tau} = D \frac{\partial^2 c}{\partial z^2}. \tag{15}$$

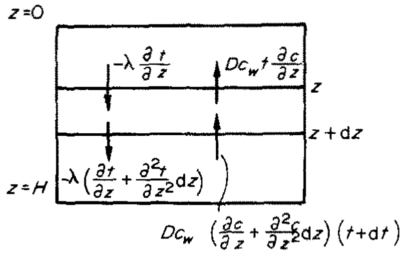

FIG. 15. Sketch for the heat balance in the moist body.

Considering Fig. 15 we can write the heat balance in a similar way:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}(c_{na}\rho_{na}t\,\mathrm{d}z) = \left[c_W D\left(\frac{\partial c}{\partial z}+\frac{\partial^2 c}{\partial z^2}\mathrm{d}z\right)\left(t+\frac{\partial t}{\partial z}\mathrm{d}z\right) - \lambda_{na}\frac{\partial t}{\partial z}\right] - \left[c_W D\frac{\partial c}{\partial z}t - \lambda_{na}\left(\frac{\partial t}{\partial z}+\frac{\partial^2 t}{\partial z^2}\mathrm{d}z\right)\right],$$

and

$$c_{na}\rho_{na}\frac{\partial t}{\partial \tau} = c_W D\frac{\partial^2 c}{\partial z^2}t + c_W D\frac{\partial c}{\partial z}\frac{\partial t}{\partial z} + \lambda_{na}\frac{\partial^2 z}{\partial z^2},$$

so

$$\frac{\partial t}{\partial \tau} = \frac{1}{c_{na}\rho_{na}}\left[c_W D\left(\frac{\partial^2 c}{\partial z^2}t+\frac{\partial c}{\partial z}\frac{\partial t}{\partial z}\right)+\lambda_{na}\frac{\partial^2 t}{\partial z^2}\right]. \quad (16)$$

The initial and boundary conditions

Initial conditions:

$$\begin{aligned} t(0,z) &= t_0, \\ c(0,z) &= c_0. \end{aligned} \quad (17)$$

Boundary conditions. At first we investigate the heat and mass transport at the surface of moist body. Because there is not source or sink in the boundary layer the total mass of liquid arised by diffusion to the surface from the moist body evaporates into the drying medium:

$$D\frac{\partial c}{\partial z}(\tau, 0) = N. \quad (18)$$

Similar boundary condition can be written in connection the heat transport, too. One part of the heat flux makes the moisture diffused to the surface evaporate and the other one raises the temperature of the moist body:

$$q_s - Nr_F = -\lambda_{na}\frac{\partial t}{\partial z}(\tau, 0). \quad (19)$$

It is assumed here that the moisture is diffusing in a liquid state to the surface and the total evaporation is achieved there. In order to consider further boundary conditions we assume there is not heat or mass transfer on the opposite surface of the moist body or they may be rather neglected.

So we can write:

$$-\lambda_{na}\frac{\partial t}{\partial z}(\tau, H) = 0$$

and

$$D\frac{\partial c}{\partial z}(\tau, H) = 0,$$

or rather

$$\begin{aligned} \frac{\partial t}{\partial z}(\tau, H) &= 0, \\ \frac{\partial c}{\partial z}(\tau, H) &= 0. \end{aligned} \quad (20)$$

as well as

$$x_F = \varphi(t_F, c) \quad (21)$$

where the last equation represents the sorption property of the moist body. In order to facilitate the managing of these equations we turn to dimensionless variables.

For this sake, we define the following dimensionless groups:

$$\xi = \frac{c}{c_0},$$

$$\upsilon = \frac{t}{t_0}$$

$$Fo = \frac{a_{na}\tau}{H^2},$$

$$Bi = \frac{\alpha . z}{\lambda_{na}} \quad \text{and} \quad Bi(H) = \overline{Bi}$$

$$Le_{na} = \frac{a_{na}}{D} = \frac{1}{Lu},$$

$$Le_L = \frac{a_L}{D_L},$$

$$Le_0 = \frac{\lambda_{na}}{c_0 D c_W}.$$

The differential equation system describing the drying process in dimensionless form is the following:

$$\frac{\partial \xi}{\partial F_0}(F_0, Bi) = \frac{\overline{Bi}^2}{Le_{na}}\frac{\partial^2 \xi}{\partial Bi^2} \quad (15')$$

$$\frac{\partial \upsilon}{\partial F_0}(F_0, Bi) = \overline{Bi}^2\left\{\frac{1}{Le_0}\left[\frac{\partial^2 \xi}{Bi}\upsilon(F_0, Bi) + \frac{\partial \xi}{\partial Bi}\frac{\partial \upsilon}{\partial Bi}\right] + \frac{\partial^2 \upsilon}{\partial Bi^2}\right\} \quad (16')$$

$$x_F = x_F[\upsilon(F_0, 0), \xi]. \quad (21')$$

The boundary conditions:

$$-\frac{\partial \upsilon}{\partial Bi}(F_0, 0) = [\upsilon_L - \upsilon(F_0, 0)] - \frac{1}{Le_L^{\xi}} \times \left[\frac{r_0}{t_0 c_{nL}} + \frac{c_{PWG}}{c_{nL}} \upsilon_L - \frac{c_W}{c_{nL}} \upsilon(F_0, 0)\right](x_F - x_L) \quad (19')$$

$$\frac{\partial \xi}{\partial Bi}(F_0, 0) = \frac{Le_0}{Le_L^{\xi}} \frac{c_W}{c_{nL}}(x_F - x_L), \quad (18')$$

$$\frac{\partial \upsilon}{\partial Bi}(F_0, \overline{Bi}) = 0$$
$$\frac{\partial \xi}{\partial Bi}(F_0, \overline{Bi}) = 0. \quad (20')$$

The initial conditions:

$$\upsilon(0, Bi) = 1,$$
$$\xi(0, Bi) = 1. \quad (17')$$

The excepted values of the variables are:

$$0 \leqq \xi, \quad F_0 < +\infty, \quad \text{and} \quad 1 \leqq \upsilon \leqq +\infty,$$
$$0 \leqq Bi = \overline{Bi} < +\infty.$$

Solution of the nonlinear boundary value problem

To obtain solution to this nonlinear boundary value problem, (15)–(21) the finite difference method can be used. But to begin the integration we must know the values of $\upsilon(F_0, 0)$ and $\xi(F_0, 0)$. There is possibility to solve this problem by iteration. We respect two approaching values of them and after the procedure we control whether the conditions (20) are satisfied at

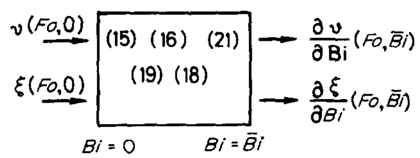

FIG. 16. A representation for the iterative method.

$Bi = \overline{Bi}$. The idea is represented by Fig. 16. We can define the following functions in order to apply the Newton–Raphson method.

$$f(x, y) \triangleq \frac{\partial \xi}{\partial Bi}(F_0, \overline{Bi}), \quad (22)$$

$$g(x, y) \triangleq \frac{\partial \upsilon}{\partial Bi}(F_0, \overline{Bi}), \quad (23)$$

where

$$x \equiv \xi(F_0, 0),$$
$$y \equiv \upsilon(F_0, 0).$$

In this case the problem can be considered as the determination of the set which is constituted by the following elements:

$$\Omega = \{x, y : f(x, y) = g(x, y) = 0 \quad \text{and} \quad 0 \leqslant F_0 < +\infty\}.$$

Let us consider x_j and y_j as an approach of the solution at fixed $F_0 = (F_0)_k$, using the Newton–Raphson method we are interested in the solution of the following equations:

$$v\frac{\partial f}{\partial x}(x_j, y_j) + s\frac{\partial f}{\partial y}(x_j, y_j) + f(x_j, y_j) = 0,$$
$$v\frac{\partial g}{\partial x}(x_j, y_j) + s\frac{\partial g}{\partial y}(x_j, y_j) + g(x_j, y_j) = 0$$

if

$$D(x_j, y_j) \neq 0 \qquad j = 1, 2, \ldots, n$$

and

$$D = \begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix}$$

where n is the number of the iterative steps, s and v can be calculated and the next approach is:

$$x_{j+1} = x_j + v$$
$$y_{j+1} = y_j + s.$$

If x_1 and y_1 are sufficiently near to the solution x_n, y_n the convergence will be rapid.

Because the functions f and g are unknown in analytical way we must calculate their derivates in numerical way, e.g.

$$\frac{\partial f}{\partial x}(x_j, y_j) \approx \frac{f(x_j + \Delta, y_j) - f(x_j, y_j)}{\Delta}$$

$$\frac{\partial f}{\partial y}(x_j, y_j) \approx \frac{f(x_j, y_j + \Delta) - f(x_j, y_j)}{\Delta}$$

where Δ is a sufficiently determined length of the step.

Numerical solution

The procedure of the calculation is perceptible on Fig. 17. After choosing the initial value of $\upsilon_F = \upsilon_F(F_0, 0) = \upsilon_{0,j}$ for the first step of the iteration we can obtain $x_{F,j}$ from (21).

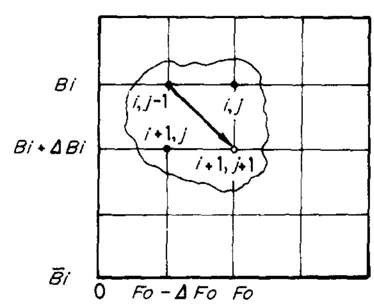

FIG. 17. Show of the integration net for finite-difference method.

Considering the equation (19) we can write:

$$-\left(\frac{\partial \upsilon}{\partial Bi}\right)_{0,j} = (\upsilon_L - \upsilon_{0,j}) - \left(\frac{r_0}{t_0 c_{nL}} + \frac{c_{PWG}}{c_{nL}}\upsilon_L - \frac{c_W}{c_{nL}}\upsilon_{0,j}\right)(x_{F,j} - x_L),$$

from which $(\partial\upsilon/\partial Bi)_{0,j}$ can be calculated. Considering the derivates in time:

$$\left(\frac{\partial \upsilon}{\partial F_0}\right)_{i,j} = \frac{\upsilon_{i,j} - \upsilon_{i,j-1}}{\Delta F_0},$$

$$\left(\frac{\partial \xi}{\partial F_0}\right)_{i,j} = \frac{\xi_{i,j} - \xi_{i,j-1}}{\Delta F_0} \qquad i, j = 0, \ldots, N, M.$$

Knowing the concentration derivates in time and considering (15):

$$\left(\frac{\partial^2 \xi}{\partial Bi^2}\right)_{i,j} = \frac{Le_{na}}{\overline{Bi}^2}\left(\frac{\partial \xi}{\partial F_0}\right)_{i,j}$$

and applying the equation (16)

$$\left(\frac{\partial^2 \upsilon}{\partial Bi^2}\right)_{i,j} = \frac{1}{\overline{Bi}^2}\left(\frac{\partial \upsilon}{\partial F_0}\right)_{i,j} - \frac{1}{Le_0} \times \left[\left(\frac{\partial^2 \xi}{\partial Bi^2}\right)_{i,j}\upsilon_{i,j} + \left(\frac{\partial \xi}{\partial Bi}\right)_{i,j}\left(\frac{\partial \upsilon}{\partial Bi}\right)_{i,j}\right]$$

so

$$\left(\frac{\partial \xi}{\partial Bi}\right)_{i+1,j} = \left(\frac{\partial \xi}{\partial Bi}\right)_{i,j} + \Delta Bi\left(\frac{\partial^2 \xi}{\partial Bi^2}\right)_{i,j},$$

and

$$\left(\frac{\partial \upsilon}{\partial Bi}\right)_{i+1,j} = \left(\frac{\partial \upsilon}{\partial Bi}\right)_{i,j} + \Delta Bi\left(\frac{\partial^2 \upsilon}{\partial Bi^2}\right)_{i,j},$$

can be calculated, too.

Afterwards the values of the functions are:

$$\upsilon_{i+1,j} = \upsilon_{i,j} + \Delta Bi\left(\frac{\partial \upsilon}{\partial Bi}\right)_{i,j},$$

$$\xi_{i+1,j} = \xi_{i,j} + \Delta Bi\left(\frac{\partial \upsilon}{\partial Bi}\right)_{i,j}.$$

We have to choose in advance the value of $\xi_{0,j}$ in similar way as the value of the temperature of the surface.

In this way we go on this procedure from $i = 0$ to $i = N$. We can control whether the initial values of the iteration were correctly chosen.

$$\left|\frac{\partial \xi}{\partial Bi_{N,j}} + \frac{\partial \upsilon}{\partial Bi_{N,j}}\right| < \varepsilon$$

where $\varepsilon > 0$ is a chosen limit of the error. If this condition is satisfied we can repeat this procedure starting at $0, j+1$ increasing the value of j to $j+1$ and of course choosing the values of $\upsilon_{0,j+1}$ and $\xi_{0,j+1}$. If this condition is not satisfied we can use the iteration method described in the later point. The dates of the example illustrating this model are the following:

$$\begin{aligned} z &= 1 \\ Le_L &= 0{\cdot}60 \\ Le_0 &= 0{\cdot}4 \\ Le_{na} &= 0{\cdot}333; \quad Lu = 3; \\ \overline{Bi}_q &= 0{\cdot}2 \\ c_W &= 1\ \mathrm{kcal/kg\,^\circ C} \\ c_{nL} &= 0{\cdot}24\ \mathrm{kcal/kg\,^\circ C} \\ c_{PWG} &= 0{\cdot}46\ \mathrm{kcal/kg\,^\circ C} \\ t_0 &= 20\,^\circ\mathrm{C} \\ r_0 &= 597\ \mathrm{kcal/kg} \\ \upsilon_L &= 6. \end{aligned}$$

In connection with the relation (21) we supposed that the partial vapor tension of the moisture could be expressed in the following way:

$$p_F = p_{tel}\frac{c}{c+50} \tag{21}$$

where

$$p_{tel} = f(t_F).$$

The results of the calculation are represented by Figs. 18–19. On Figs. 18 and 19 we see the distribution of the dimensionless temperature and concentration depending on the Fourier number and using constant Biot-number as parameter. The figures show clearly

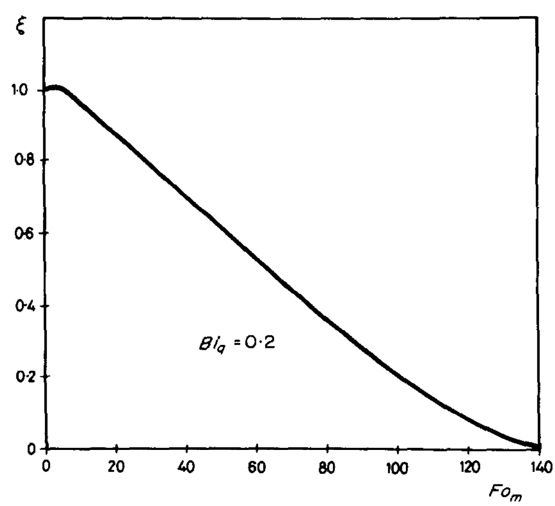

FIG. 18. Dimensionless moisture content vs. Fourier number.

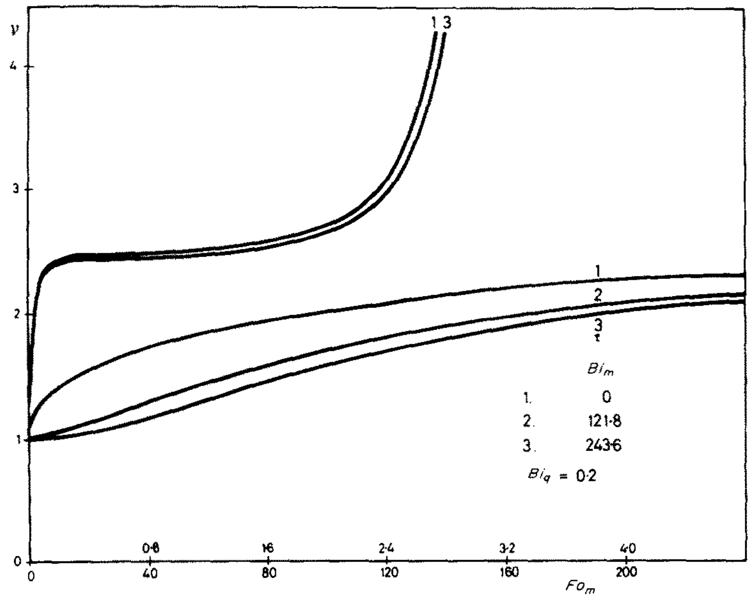

FIG. 19. Dimensionless temperature vs. Fourier number.

that the concentration gradient in the cross-section has low values caused by the high value of the coefficient of the moisture diffusivity in opposite of the thermal gradient. It appears from the figures that the temperature becomes nearly constant named the wet bulb temperature in the different depth of the moist body. This temperature is reached by the different layers in different time and later their temperatures are separated again. This kind of the temperature curves were observed at the drying of sand [3].

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ETUDE DU SECHAGE DISCONTINU

Résumé—Cet article propose deux modèles mathématiques qui permettent l'étude du mécanisme de séchage discontinu. La première partie concerne l'influence du changement des propriétés du milieu et la seconde considère l'influence des gradients dans la section transversale. Les modèles proposés sont utilisables pour l'étude du séchage de plaques finies avec des propriétés de sorption variables et non linéaires. Les solutions numériques sont illustrées par des exemples calculés sur ordinateur.

UNTERSUCHUNG ÜBER AUSTROCKNUNG

Zusammenfassung—Es werden zwei mathematische Modelle für die Untersuchung der Austrocknung vorgeschlagen. Im ersten wird der Einfluß der variablen Eigenschaften des zu trocknenden Mediums und im zweiten der Einfluß der Gradienten über den Querschnitt berücksichtigt. Die vorgeschlagenen Modelle sind geeignet zur Untersuchung der Trocknung endlicher Platten bei wechselnden und nichtlinearisierten Sorptions-Eigenschaften. Die numerische Lösung wird durch Beispiele, die mit dem Computer gelöst werden, veranschaulicht.

ИССЛЕДОВАНИЕ ПЕРИОДИЧЕСКОЙ СУШКИ

Аннотация — В статье предложены две математические модели, которые позволяют исследовать механизм периодической сушки. В первой части изучается влияние изменения свойств теплоносителя, во второй части расматривается влияние градиентов в поперечном сечении. Предлагаемые модели могут применяться для исследования сушки ограниченной пластины при переменных и нелинеаризованных сорбционных свойствах. Численное решение получено на вычислительной машине.